

## Solutions

## 4.1 Exponential Models

4.1.1. Find the value of  $P(t) = 500(1.04)^t$  at  $t = 2$ .

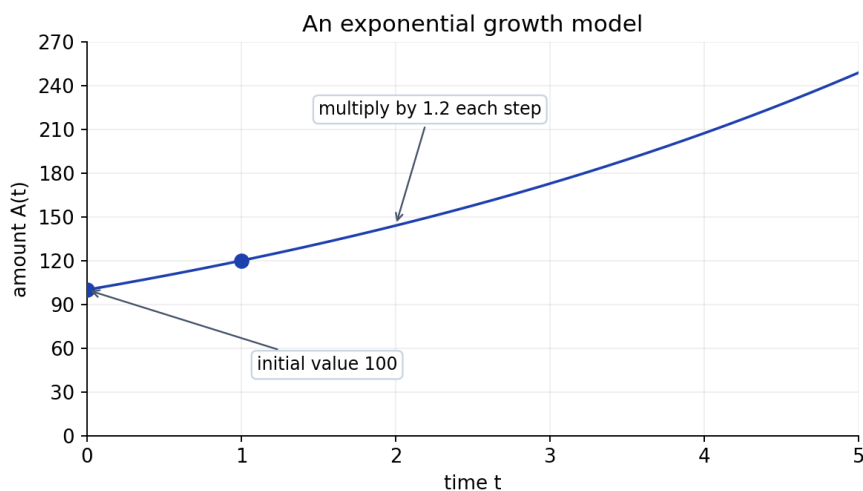
Solution.  $P(2) = 500(1.04)^2 = 540.8$ .

4.1.2. A quantity halves every 3 hours. Write its model when it starts at 120.

Solution. The number of half-life intervals is  $t/3$ , so  $A(t) = 120(1/2)^{t/3}$ .

4.1.3. From the graph of  $A(t) = 100(1.2)^t$ , identify the initial value and percentage growth per time unit.

Solution.  $A(0) = 100$ , so the initial value is 100. The factor  $1.2 = 1 + 0.2$  gives 20% growth per time unit.



The model starts at 100 and grows by 20% per time unit.

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## 4.2 Logarithm Properties

4.2.1. Condense  $\log_2 x + \log_2 (x - 1)$  and state the original domain.

Solution. The result is  $\log_2 (x(x - 1))$ , retaining  $x > 1$  because both original arguments must be positive.

4.2.2. Evaluate  $\log_3 (1/27)$ .

Solution.  $1/27 = 3^{-3}$ , so the value is  $-3$ .

4.2.3. Is  $\ln(x + 2) = \ln x + \ln 2$  a valid identity for  $x > 0$ ?

Solution. No. The product rule applies to  $2x$ , not  $x + 2$ . For example,  $x = 1$  gives  $\ln 3$  on the left and  $\ln 2$  on the right.

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## 4.3 Solving and Domains

4.3.1. Solve  $3^{2x} = 27$ .

Solution.  $27 = 3^3$ , so  $2x = 3$  and  $x = 3/2$ .

4.3.2. Solve  $\log_2(x - 3) = 4$ .

Solution. The domain requires  $x > 3$ . Rewrite as  $x - 3 = 16$ , giving  $x = 19$ .

4.3.3. Solve  $\ln x + \ln(x - 3) = \ln 4$  and reject invalid candidates.

Solution. The original domain is  $x > 3$ . Combine logs to get  $x(x - 3) = 4$ , so  $(x - 4)(x + 1) = 0$ . Only  $x = 4$  remains.