

Solutions

5.1 Arithmetic and Geometric Sequences

5.1.1. Find term 8 of $10, 7, 4, \dots$.

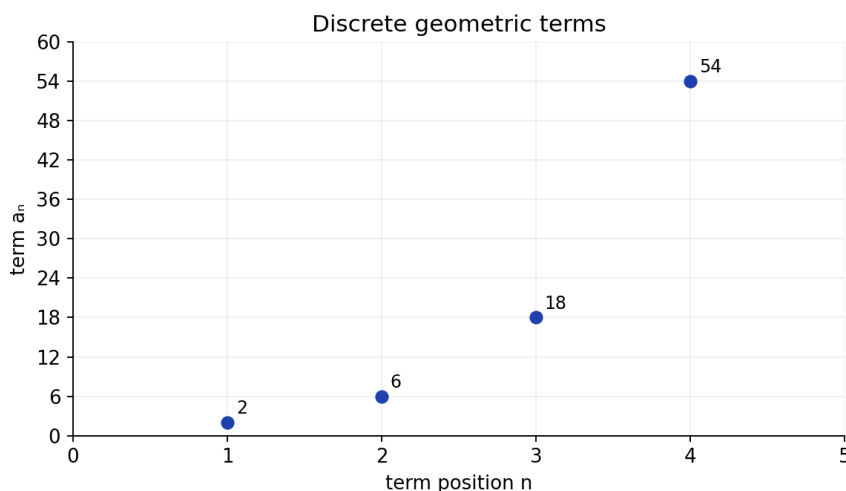
Solution. The difference is -3 , so $a_8 = 10 + 7(-3) = -11$.

5.1.2. Find term 4 of $81, 27, 9, \dots$.

Solution. The ratio is $1/3$. Thus $a_4 = 81(1/3)^3 = 3$.

5.1.3. The graph shows terms of $a_n = 2 \cdot 3^{n-1}$. Give its first four terms and explain why the graph uses separate points.

Solution. The terms are 2, 6, 18, 54. Only integer positions $n = 1, 2, 3, \dots$ belong to the sequence, so the graph is discrete.



The common ratio is 3; the first four terms are 2, 6, 18, 54.

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5.2 Explicit and Recursive Rules

5.2.1. Write a recursive rule for 2, 6, 18, ...

Solution. $a_1 = 2$ and $a_n = 3a_{n-1}$ for $n \geq 2$.

5.2.2. Write an explicit rule for $a_1 = 7$, $a_n = a_{n-1} - 2$.

Solution. The arithmetic difference is -2 , so $a_n = 7 - 2(n - 1)$.

5.2.3. Given $b_1 = 3$, $b_n = 2b_{n-1} + 1$, find b_2, b_3, b_4 .

Solution. Apply the rule sequentially: $b_2 = 7$, $b_3 = 15$, and $b_4 = 31$.

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5.3 Series and Convergence

5.3.1. Find the sum of the first 10 terms of $3, 5, 7, \dots$.

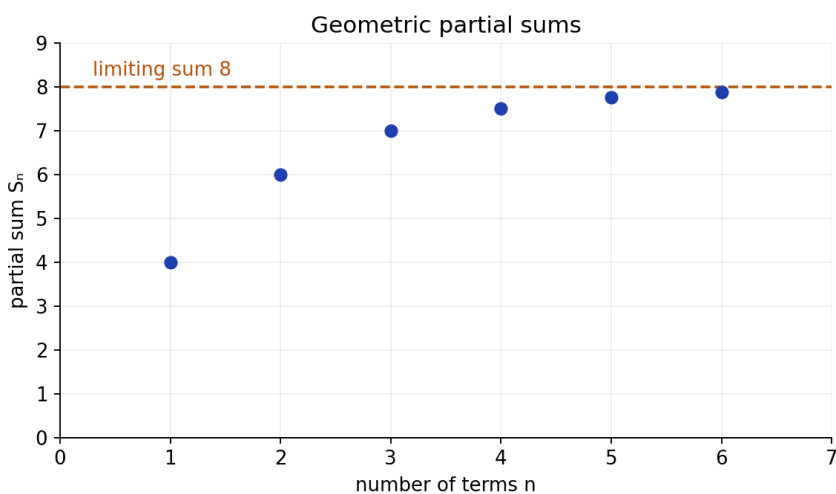
Solution. The tenth term is $3 + 9(2) = 21$. Thus $S_{10} = 10(3 + 21)/2 = 120$.

5.3.2. Does $1 + 2 + 4 + 8 + \dots$ have a finite sum?

Solution. No. The ratio is 2, so $|r| \geq 1$ and the infinite geometric series diverges.

5.3.3. The graph shows partial sums of $4 + 2 + 1 + \dots$. Find the limiting sum.

Solution. The ratio is $1/2$, so the infinite sum is $4/(1 - 1/2) = 8$. The plotted partial sums approach 8.



The infinite geometric sum is 8.