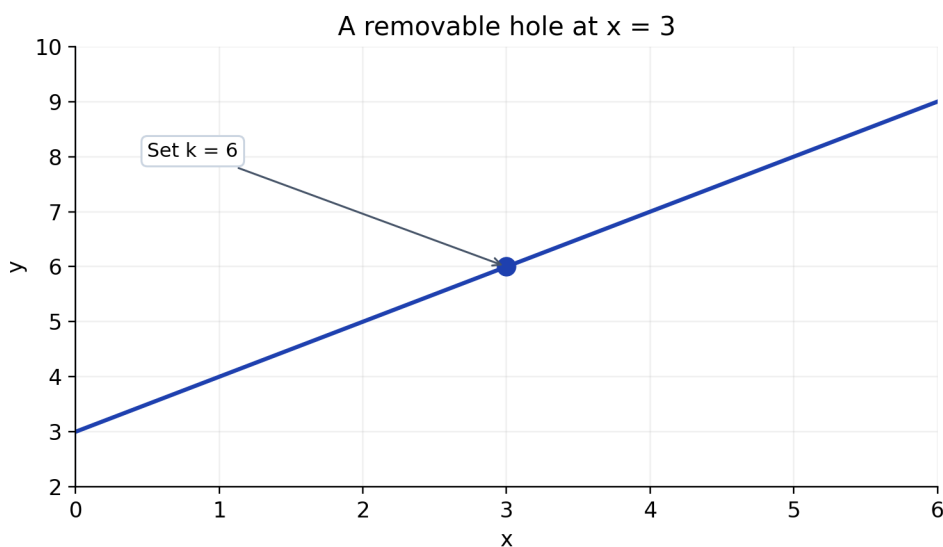


Lesson 1.4 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

1. Let $f(x) = \frac{x^2 - 9}{x - 3}$ for $x \neq 3$, and $f(3) = k$. Find k for continuity. Classify the break if $k = 0$.

Solution. For $x \neq 3$, cancellation gives $f(x) = x + 3$. Hence $\lim_{x \rightarrow 3} f(x) = 6$. Continuity requires $f(3) = 6$, so choose $k = 6$. If $k = 0$, the finite limit differs from the point value, so the discontinuity is removable.



Filling the hole at height 6 makes the function continuous.

2. Let $f(x) = ax + 2$ for $x < 1$, and $f(x) = x^2 + 4$ for $x \geq 1$. Find a so that f is continuous at 1.

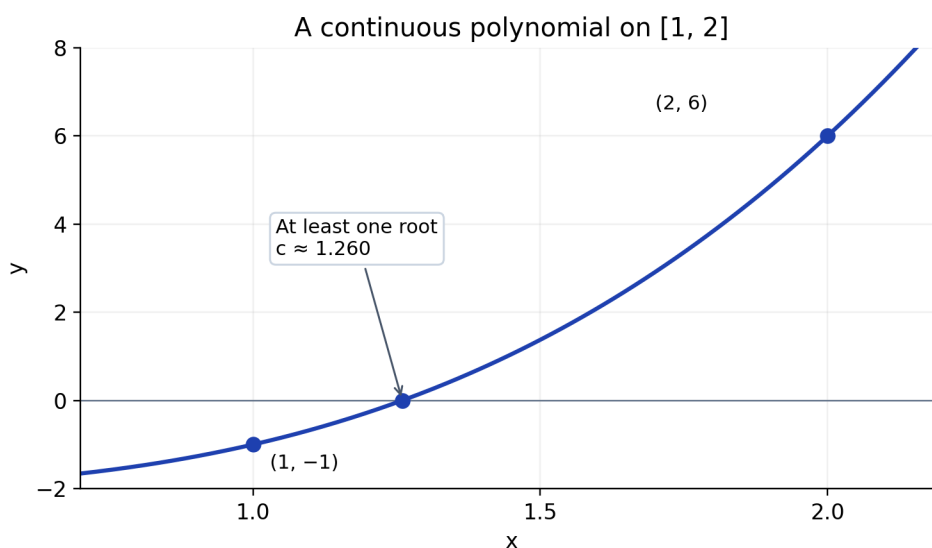
Solution. The left-hand limit is $a + 2$. The right-hand limit and $f(1)$ are both 5. Set $a + 2 = 5$, giving $a = 3$. The two one-sided limits and the point value then agree.

Lesson 1.4 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

3. Use the Intermediate Value Theorem to justify a root of $f(x) = x^3 - 2$ between 1 and 2. Does IVT alone prove uniqueness?

Solution. The polynomial is continuous on $[1, 2]$. Since $f(1) = -1$ and $f(2) = 6$, we have $-1 < 0 < 6$. IVT guarantees at least one $c \in (1, 2)$ with $f(c) = 0$. IVT alone does not establish uniqueness.



The endpoint values bracket zero. IVT guarantees a crossing; the marked root is approximate.

4. Explain why $\sqrt{x+2}$ is continuous on $[-2, 2]$. Then explain why IVT cannot be applied to $\frac{1}{x-1}$ on $[0, 2]$.

Solution. The square-root function is continuous in the interior, and its endpoint limits agree with its values: $\lim_{x \rightarrow -2^+} \sqrt{x+2} = 0$ and $\lim_{x \rightarrow 2^-} \sqrt{x+2} = 2$. In contrast, $\frac{1}{x-1}$ is undefined at 1, so it is not continuous on $[0, 2]$. The IVT hypothesis fails.