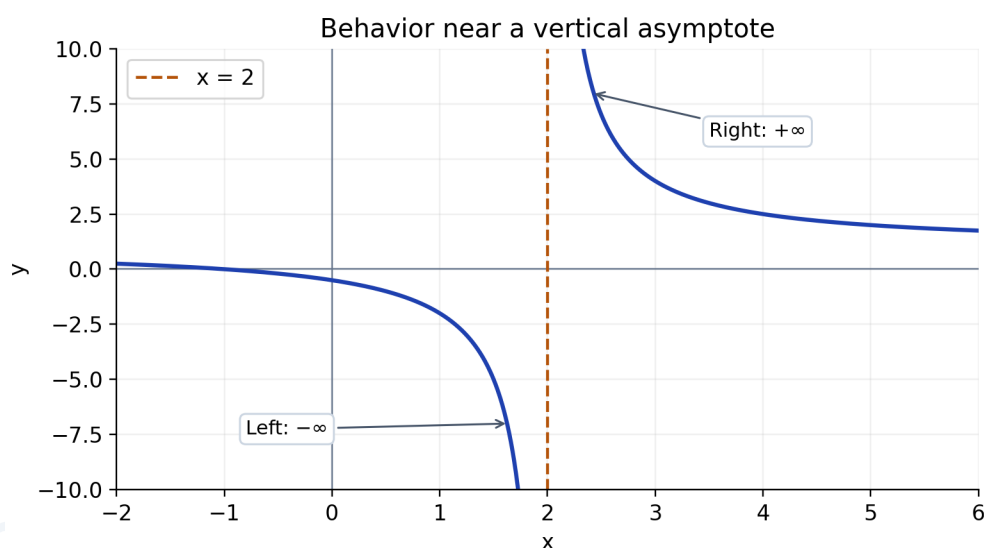


Lesson 1.5 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

1. Find $\lim_{x \rightarrow 2^-} \frac{x+1}{x-2}$ and $\lim_{x \rightarrow 2^+} \frac{x+1}{x-2}$. Does the two-sided limit exist? Identify the vertical asymptote.

Solution. The numerator approaches $3 > 0$. The denominator is negative to the left and positive to the right, so $\lim_{x \rightarrow 2^-} \frac{x+1}{x-2} = -\infty$ and $\lim_{x \rightarrow 2^+} \frac{x+1}{x-2} = +\infty$. The two-sided limit does not exist. The vertical asymptote is $x = 2$.



The left branch tends to negative infinity and the right branch to positive infinity.

2. Evaluate $\lim_{x \rightarrow +\infty} \frac{5x^2 - 3}{2x^2 + 7x}$. Identify the horizontal asymptote.

Solution. Divide by x^2 to obtain $\frac{5 - 3/x^2}{2 + 7/x}$. The reciprocal terms approach 0, so the limit is $\frac{5}{2}$. The horizontal asymptote is $y = \frac{5}{2}$.

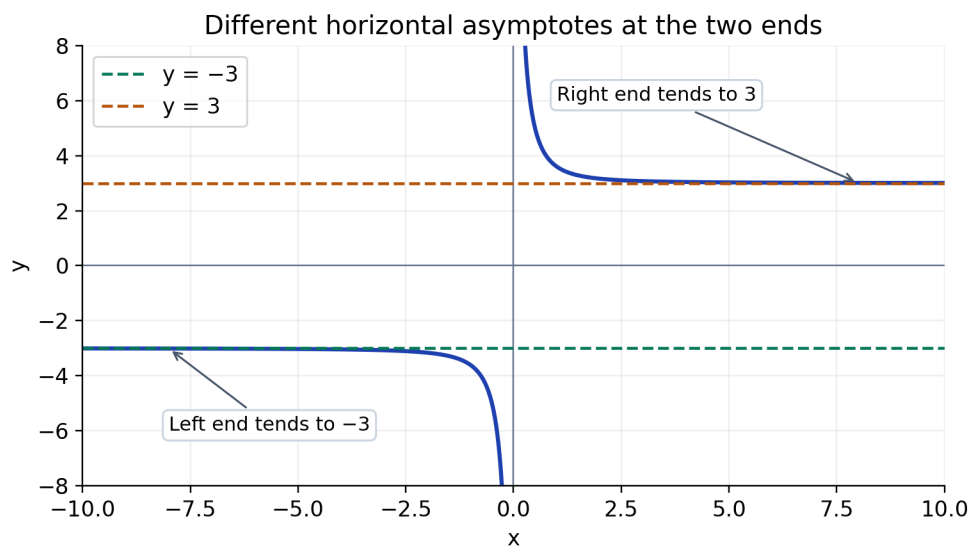
Lesson 1.5 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

3. Evaluate $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^2 + 4}}{x}$. Explain the sign.

Solution. Rewrite the quotient as $\frac{|x|}{x} \sqrt{9 + \frac{4}{x^2}}$. For negative x , $\frac{|x|}{x} = -1$, while the radical approaches 3.

Therefore the limit is -3 .



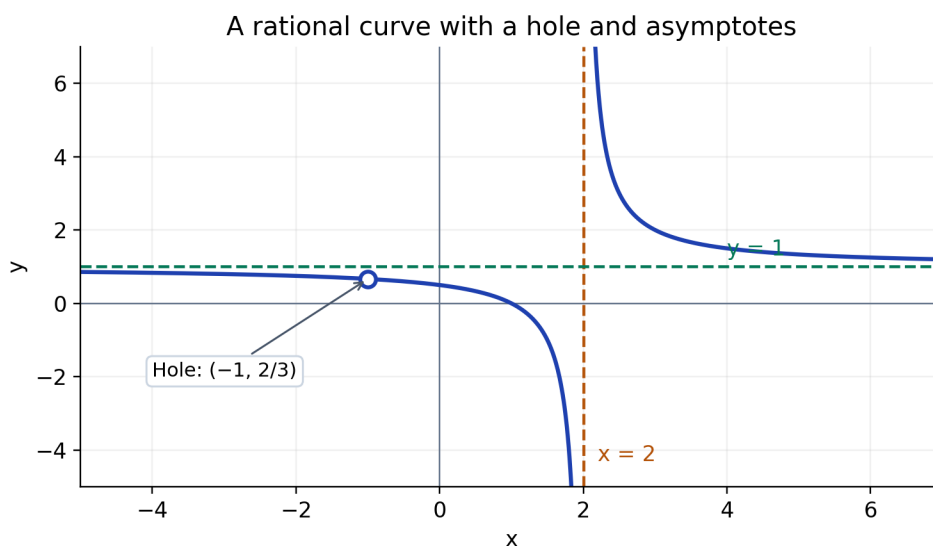
At negative infinity, the curve approaches $y = \text{minus } 3$ because absolute x over x equals minus 1.

Lesson 1.5 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

4. For $f(x) = \frac{x^2 - 1}{x^2 - x - 2}$, classify the excluded inputs and find both end limits.

Solution. Factor to get $\frac{(x-1)(x+1)}{(x-2)(x+1)}$. The excluded inputs are -1 and 2 . At -1 there is a removable hole with limit $\frac{2}{3}$. At 2 a denominator factor remains and the numerator is nonzero, giving a vertical asymptote $x = 2$. Equal leading degrees give $\lim_{x \rightarrow \pm\infty} f(x) = 1$, so $y = 1$ is a horizontal asymptote.



The hole is at (minus 1, two thirds), with asymptotes $x = 2$ and $y = 1$.