

Lesson 1.3 · Worked solutions

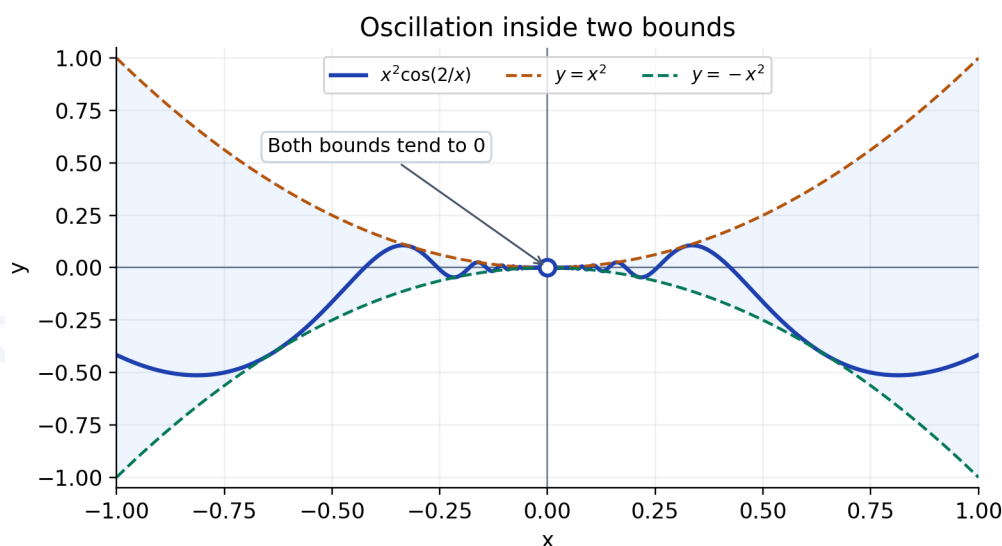
Show the method, check hypotheses, and justify your conclusion.

1. Suppose $4 - x^2 \leq f(x) \leq 4 + 2x^2$ near 0. Find $\lim_{x \rightarrow 0} f(x)$ and name the theorem.

Solution. Both bounding functions approach 4: $\lim_{x \rightarrow 0} (4 - x^2) = \lim_{x \rightarrow 0} (4 + 2x^2) = 4$. Since the inequalities hold near 0, the Squeeze Theorem gives $\lim_{x \rightarrow 0} f(x) = 4$.

2. Evaluate $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{2}{x}\right)$. Justify your answer with bounds.

Solution. Since $-1 \leq \cos(2/x) \leq 1$ and $x^2 \geq 0$, we have $-x^2 \leq x^2 \cos(2/x) \leq x^2$. Both bounds approach 0. The Squeeze Theorem gives $\lim_{x \rightarrow 0} x^2 \cos(2/x) = 0$, even though the cosine factor oscillates.



3. Evaluate $\lim_{x \rightarrow 0} \frac{\sin(6x)}{4x}$ in radians. Rewrite it using the standard trigonometric limit.

Solution. Rewrite as $\frac{6}{4} \cdot \frac{\sin(6x)}{6x}$. As $x \rightarrow 0$, the ratio approaches 1. The limit is $\frac{6}{4} = \frac{3}{2}$.

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Show the method, check hypotheses, and justify your conclusion.

4. You know only that $-1 \leq q(x) \leq 1$ near 0. Must $\lim_{x \rightarrow 0} q(x)$ exist? Give a counterexample.

Solution. No: the two bounds have different limits. The function $q(x) = \sin(1/x)$ stays between them but has no limit at 0. There are sequences of inputs tending to 0 for which the outputs stay at 1, and others for which they stay at -1 .

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