

Lesson 1.1 · Worked solutions

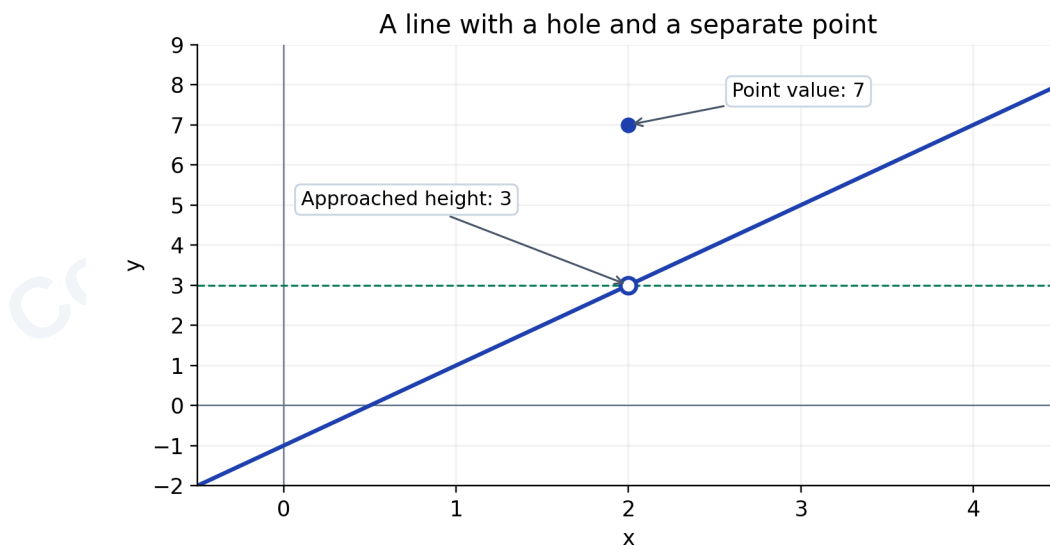
Show the method, check hypotheses, and justify your conclusion.

1. For $f(x) = \frac{x^2 - 9}{x - 3}$, $x \neq 3$, make a table at $x = 2.9, 2.99, 3.01, 3.1$. Estimate $\lim_{x \rightarrow 3} f(x)$.

Solution. The outputs are 5.9, 5.99, 6.01, 6.1. Both sides suggest 6. Factoring gives $f(x) = x + 3$ for $x \neq 3$, so $\lim_{x \rightarrow 3} f(x) = 6$.

2. Use the graph of f below. It follows $y = 2x - 1$, with an open circle at $(2, 3)$ and a filled dot at $(2, 7)$. Find $\lim_{x \rightarrow 2^-} f(x)$, $\lim_{x \rightarrow 2^+} f(x)$, $\lim_{x \rightarrow 2} f(x)$, and $f(2)$.

Solution. Tracing the line toward $x = 2$ from either side gives $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = 3$. Therefore $\lim_{x \rightarrow 2} f(x) = 3$. The filled dot gives $f(2) = 7$; its height does not change the limit.



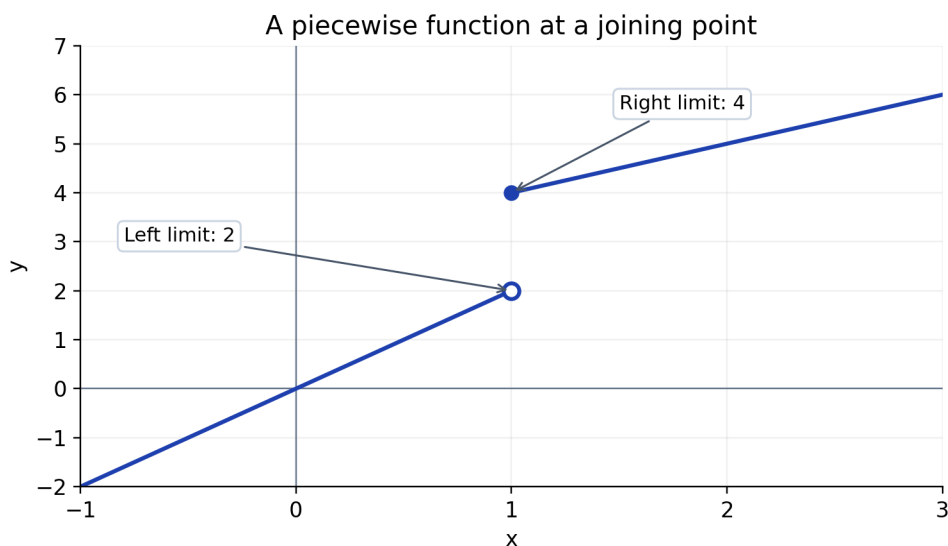
Both sides approach height 3; the filled point gives $f(2) = 7$.

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Show the method, check hypotheses, and justify your conclusion.

3. Let $p(x) = 2x$ for $x < 1$ and $p(x) = x + 3$ for $x \geq 1$. Use the graph and the formulas to find both one-sided limits at 1. Does $\lim_{x \rightarrow 1} p(x)$ exist?

Solution. The left branch approaches 2, while the right branch approaches 4. Thus $\lim_{x \rightarrow 1^-} p(x) = 2$ and $\lim_{x \rightarrow 1^+} p(x) = 4$. Since these differ, the two-sided limit does not exist, even though $p(1) = 4$.



The left and right approached heights differ, so there is no two-sided limit.

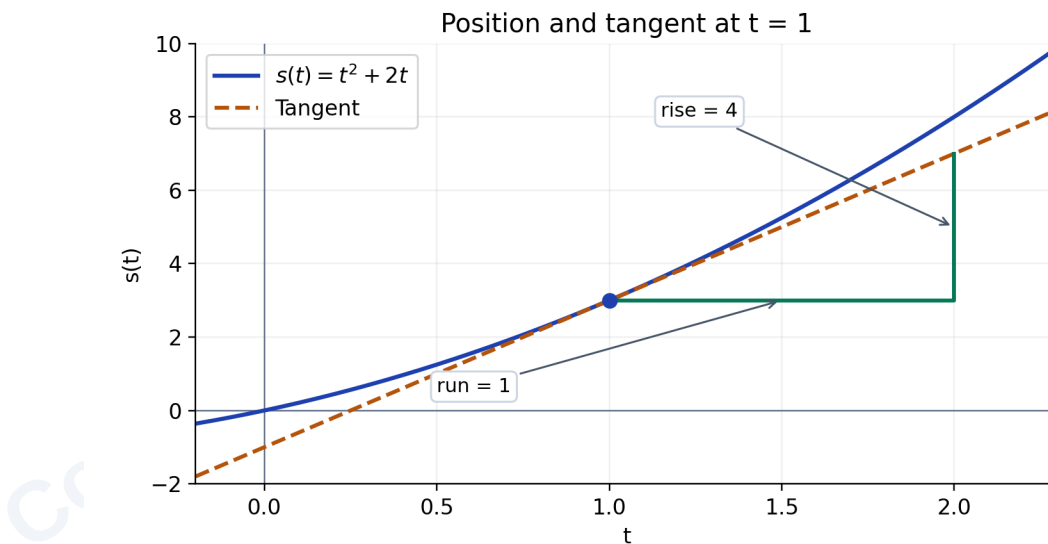
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Show the method, check hypotheses, and justify your conclusion.

4. For $s(t) = t^2 + 2t$, the average velocity on $[1, 1 + h]$ is $\frac{s(1+h) - s(1)}{h} = 4 + h$ for $h \neq 0$. Evaluate it for $h = 0.1, 0.01, -0.01$. Find the instantaneous velocity $s'(1)$.

Solution. The averages are 4.1, 4.01, 3.99. The derivative is the limit of these average rates:

$s'(1) = \lim_{h \rightarrow 0} \frac{s(1+h) - s(1)}{h} = \lim_{h \rightarrow 0} (4 + h) = 4$. The tangent line at $(1, 3)$ has slope 4, in distance units per time unit.



The tangent has rise 4 over run 1, matching the derivative s' of $1 = 4$.