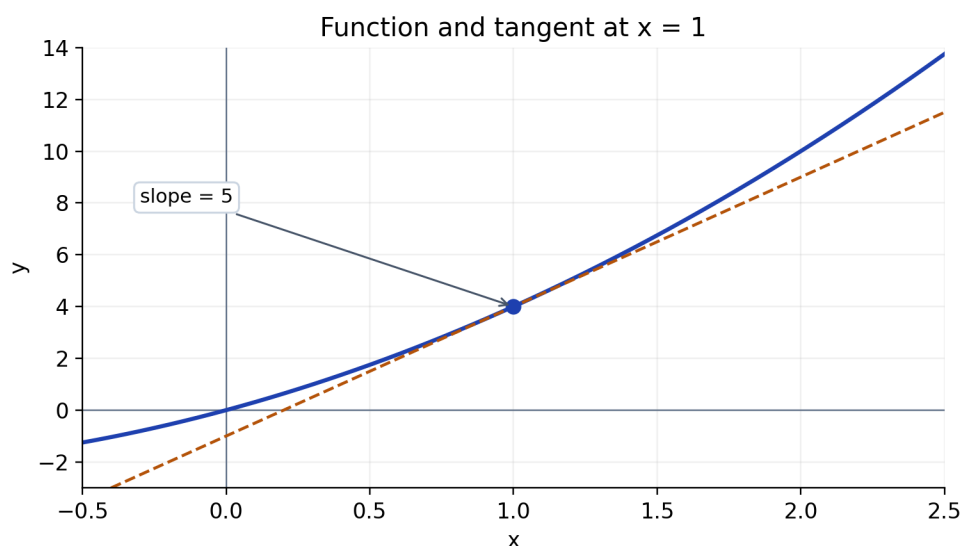


## Lesson 2.1 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

1. Use the limit definition to find  $f'(1)$  for  $f(x) = x^2 + 3x$ . Write its tangent line.

**Solution.** The difference quotient simplifies to  $5 + h$ . Its limit is 5. Since  $f(1) = 4$ , the tangent is  $y - 4 = 5(x - 1)$ , or  $y = 5x - 1$ .



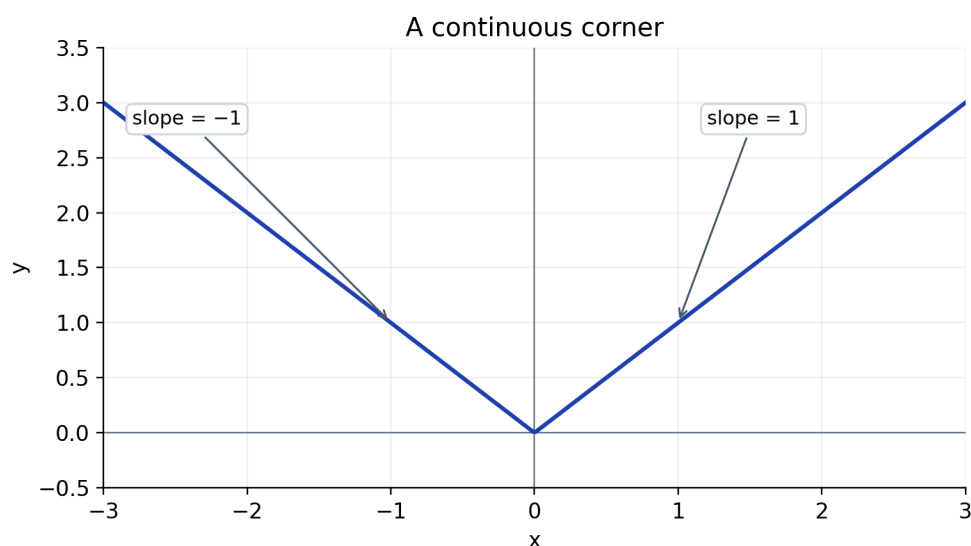
The tangent at  $(1, 4)$  is  $y = 5x$  minus 1, with slope 5.

## Lesson 2.1 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

2. Is  $f(x) = |x|$  differentiable at 0? Use one-sided slopes.

**Solution.** The left difference quotients equal  $-1$ , while the right quotients equal  $1$ . Since their limits differ,  $f'(0)$  does not exist, although  $f$  is continuous.



The left slope is minus 1 and the right slope is 1, so no derivative exists at zero.

3. Express  $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$  as a derivative and evaluate it.

**Solution.** This is  $f'(3)$  for  $f(x) = x^2$ . The quotient is  $6 + h$ , so the limit is 6.

## Lesson 2.2 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

4. Differentiate  $f(x) = 4x^3 - 5x^2 + 7$ .

**Solution.** Apply the power rule to each term:  $f'(x) = 12x^2 - 10x$ .

5. Differentiate  $g(x) = e^x + \ln x - 2\cos x$  and state the domain.

**Solution.** The domain is  $x > 0$ . The derivative is  $g'(x) = e^x + 1/x + 2\sin x$ .

6. For  $s(t) = t^3 - 6t^2 + 9t$ , find velocity and acceleration at  $t = 2$ .

**Solution.** Velocity is  $v(t) = 3t^2 - 12t + 9$ , so  $v(2) = -3$ . Acceleration is  $a(t) = 6t - 12$ , so  $a(2) = 0$ . Zero acceleration at an instant does not mean zero velocity.

## Lesson 2.3 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

7. Differentiate  $f(x) = (2x^2 + 1)^3$ .

**Solution.** The outer derivative is  $3(2x^2 + 1)^2$  and the inner derivative is  $4x$ , giving  $f'(x) = 12x(2x^2 + 1)^2$ .

8. Differentiate  $g(x) = x^2 e^{2x}$ .

**Solution.** Use product and chain rules:  $g'(x) = 2xe^{2x} + 2x^2 e^{2x}$ .

9. Find  $h'(2)$  for  $h(x) = \frac{x^2}{x+1}$ .

**Solution.** The quotient rule gives  $h'(x) = \frac{2x(x+1) - x^2}{(x+1)^2}$ . Substituting 2 gives  $\frac{8}{9}$ .

## Lesson 2.4 • Worked solutions

Show the method, check hypotheses, and justify your conclusion.

10. For  $xy + y^2 = 6$ , find  $y'$  at  $(1, 2)$ .

**Solution.** Differentiate to get  $y + xy' + 2yy' = 0$ . Thus  $y' = -y/(x + 2y)$ ; at  $(1, 2)$  the slope is  $-2/5$ .

11. If  $f(3) = 7$  and  $f'(3) = 4$ , find  $(f^{-1})'(7)$ .

**Solution.** The inverse input 7 corresponds to original input 3. Thus  $(f^{-1})'(7) = 1/f'(3) = 1/4$ .

12. Differentiate  $h(x) = \arctan(2x)$ .

**Solution.** The inner derivative is 2, so  $h'(x) = \frac{2}{1 + 4x^2}$ .