

Lesson 4.1 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

1. Find $\int(4x^3 - 2x + 1)dx$ and check it.

Solution. The family is $x^4 - x^2 + x + C$. Its derivative is $4x^3 - 2x + 1$.

2. Find $\int(2/x + e^x)dx$ on an interval excluding zero.

Solution. The result is $2\ln|x| + e^x + C$. Differentiation gives $2/x + e^x$ on either permitted side of zero.

3. Solve $s''(t) = 6t$ with $s'(0) = 2$ and $s(0) = 1$.

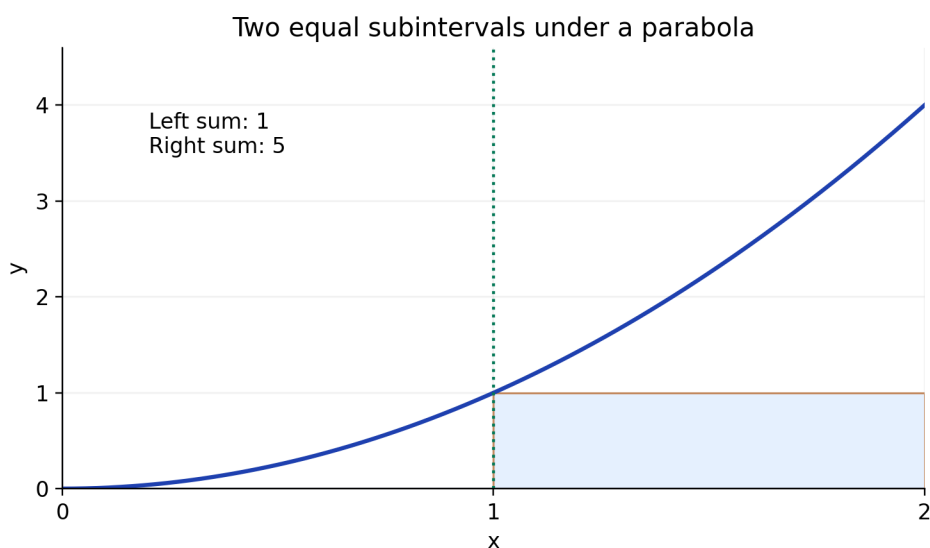
Solution. Integrate once: $s'(t) = 3t^2 + C_1$, so $C_1 = 2$. Integrate again: $s(t) = t^3 + 2t + C_2$, so $C_2 = 1$. The result is $t^3 + 2t + 1$.

Lesson 4.2 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

4. Approximate $\int_0^2 x^2 dx$ using two equal subintervals with left and right sums.

Solution. The width is 1. The left sum is $0^2 + 1^2 = 1$; the right sum is $1^2 + 2^2 = 5$. Since x^2 is increasing, these bound the exact value $8/3$.



Shaded left rectangles have total area 1; right endpoints give total area 5.

5. A table gives $f(0) = 1$, $f(1) = 3$, and $f(3) = 5$. Use the trapezoidal rule to approximate $\int_0^3 f(x) dx$.

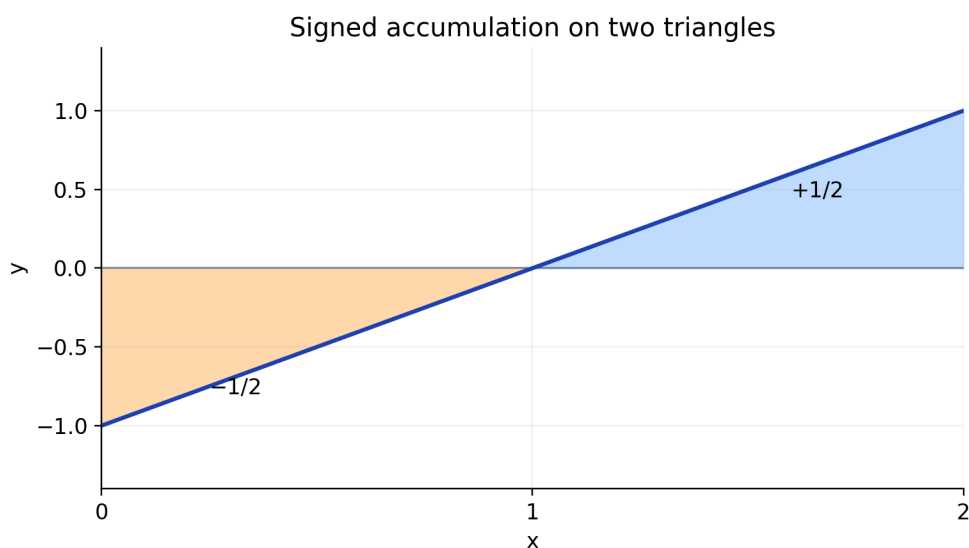
Solution. The widths are 1 and 2. The estimate is $1(1 + 3)/2 + 2(3 + 5)/2 = 10$.

Lesson 4.2 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

6. Find $\int_0^2 (x - 1) dx$ geometrically. Compare it with total geometric area.

Solution. The triangle below the axis has signed area $-1/2$ and the triangle above has area $1/2$. The signed integral is 0, while geometric area is 1.



Signed areas are minus one half and plus one half, so the integral is zero.

Lesson 4.3 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

7. Differentiate $A(x) = \int_0^{3x} (t^2 + 1) dt$.

Solution. FTC and the chain rule give $A'(x) = ((3x)^2 + 1) \cdot 3 = 27x^2 + 3$.

8. Evaluate $\int_1^3 (2x + 1) dx$.

Solution. An antiderivative is $x^2 + x$. Subtract endpoint values: $12 - 2 = 10$.

9. Differentiate $B(x) = \int_x^{x^2} e^t dt$.

Solution. The upper bound contributes $2xe^{x^2}$ and the lower bound contributes $-e^x$. Thus $B'(x) = 2xe^{x^2} - e^x$.

Lesson 4.4 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

10. Find $\int 3x^2 e^{x^3} dx$.

Solution. Use $u = x^3$ and $du = 3x^2 dx$. The result is $e^{x^3} + C$.

11. Evaluate $\int_0^1 \frac{2x}{x^2 + 1} dx$.

Solution. Set $u = x^2 + 1$ with bounds 1 and 2. The result is $[\ln u]_1^2 = \ln 2$.

12. Find $\int \frac{1}{4 + x^2} dx$ and verify the coefficient.

Solution. The result is $\frac{1}{2} \arctan(x/2) + C$. Differentiation gives $\frac{1}{2} \cdot \frac{1/2}{1 + x^2/4} = 1/(4 + x^2)$.