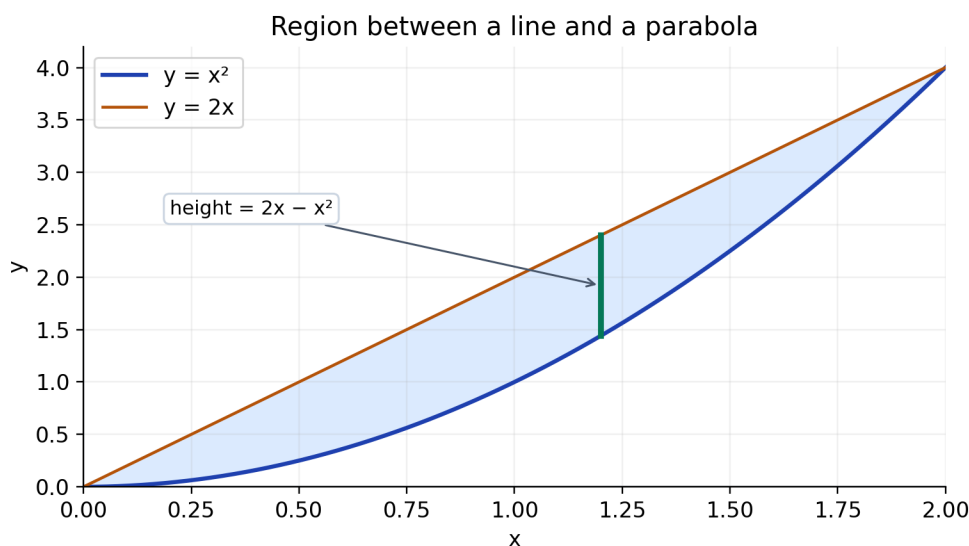


Lesson 5.1 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

1. Find the area enclosed by $y = 2x$ and $y = x^2$.

Solution. The intersections are 0 and 2, and the line is above. The area is $\int_0^2 (2x - x^2) dx = 4/3$.



The line is above the parabola; integrate $2x$ minus x squared from 0 to 2.

2. Find the average value of $f(x) = 3x^2$ on $[0, 2]$.

Solution. The integral is $[x^3]_0^2 = 8$. Divide by width 2 to obtain average value 4.

3. Find the geometric area between $y = x$ and the x-axis on $[-2, 1]$.

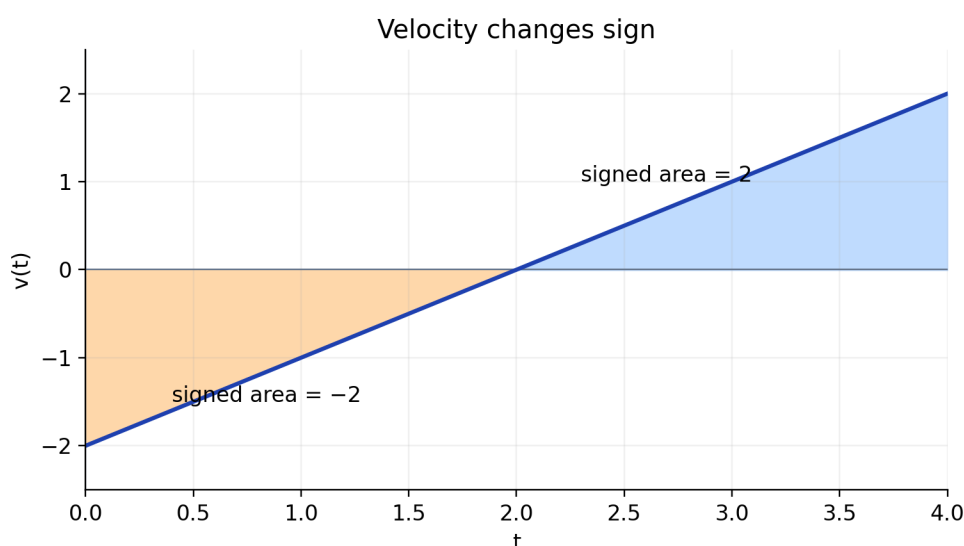
Solution. Split at 0: $\int_{-2}^0 (-x) dx + \int_0^1 x dx = 2 + 1/2 = 5/2$. The signed integral would instead be $-3/2$.

Lesson 5.2 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

4. For $v(t) = t - 2$ on $[0, 4]$, find displacement and total distance.

Solution. The signed triangles cancel, so displacement is 0. Each triangle has area 2, so total distance is 4.



The negative and positive areas cancel for displacement, but add to give distance 4.

5. A tank starts at 10 L and has net rate $3 - t$ L/min on $[0, 4]$. Find its amount at 4.

Solution. Net change is $[3t - t^2/2]_0^4 = 4$. The amount is therefore 14 liters.

6. With the same tank data, when is the amount greatest on $[0, 4]$, and what is it?

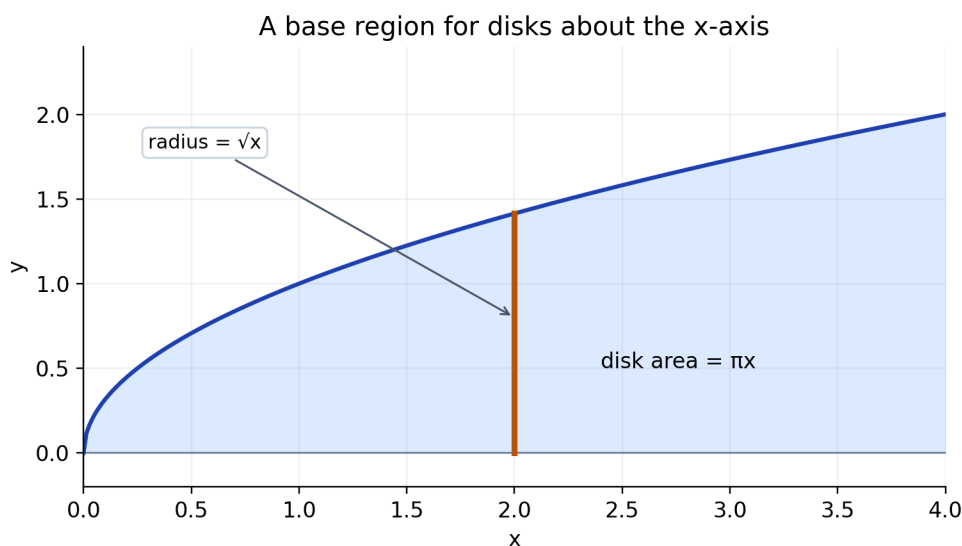
Solution. The net rate changes from positive to negative at $t = 3$. The amount there is $10 + \int_0^3 (3 - t) dt = 29/2$ liters, exceeding endpoint amounts 10 and 14.

Lesson 5.3 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

7. Rotate the region under $y = \sqrt{x}$ on $[0, 4]$ about the x-axis. Find the volume.

Solution. The disk radius is $\sqrt{x}$. Thus $V = \pi \int_0^4 x dx = 8\pi$.



The radius is square root of x , so disk area is π times x .

8. Rotate the region between $y = 3$ and $y = x$ on $[0, 3]$ about the x-axis.

Solution. The washer radii are 3 and x , giving $V = \pi \int_0^3 (9 - x^2) dx = 18\pi$.

9. Rotate the region between $y = 2$ and $y = x$ on $[0, 2]$ about $y = 2$.

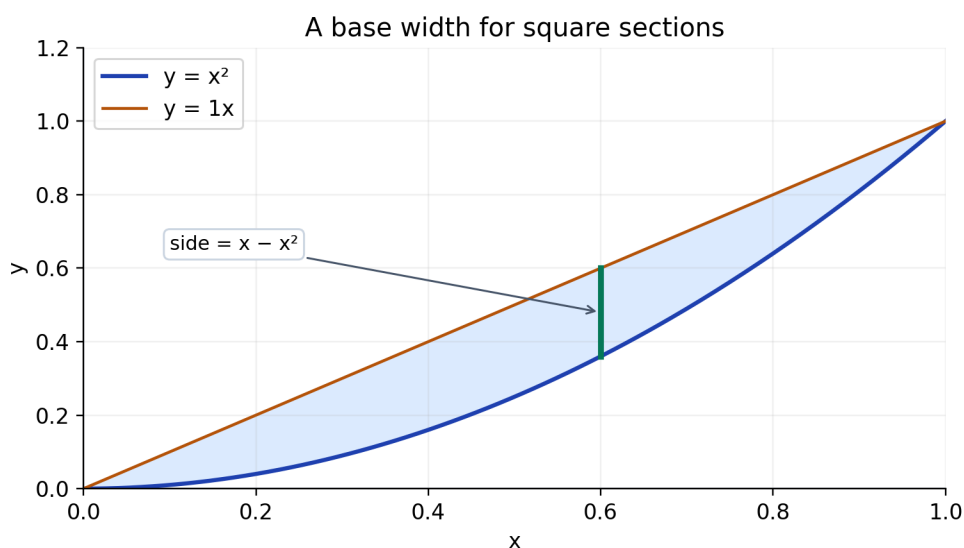
Solution. Each cross section is a disk with radius $2 - x$. Thus $V = \pi \int_0^2 (2 - x)^2 dx = 8\pi/3$.

Lesson 5.4 · Worked solutions

Show the method, check hypotheses, and justify your conclusion.

10. The base is between $y = x$ and $y = x^2$ on $[0, 1]$. Perpendicular square sections have a side equal to the base width. Find the volume.

Solution. The width is $x - x^2$. Thus $V = \int_0^1 (x - x^2)^2 dx = 1/3 - 1/2 + 1/5 = 1/30$.



The side is x minus x squared; cross-sectional area is its square.

11. For the same base, use semicircular sections with diameter equal to the width. Find the volume.

Solution. The section area is $\pi(x - x^2)^2/8$. Therefore $V = (\pi/8)(1/30) = \pi/240$.

12. A solid has triangular sections with base x and height $2x$ for $0 \leq x \leq 3$. Find its volume.

Solution. The section area is $(1/2)x(2x) = x^2$. Hence $V = \int_0^3 x^2 dx = 9$.