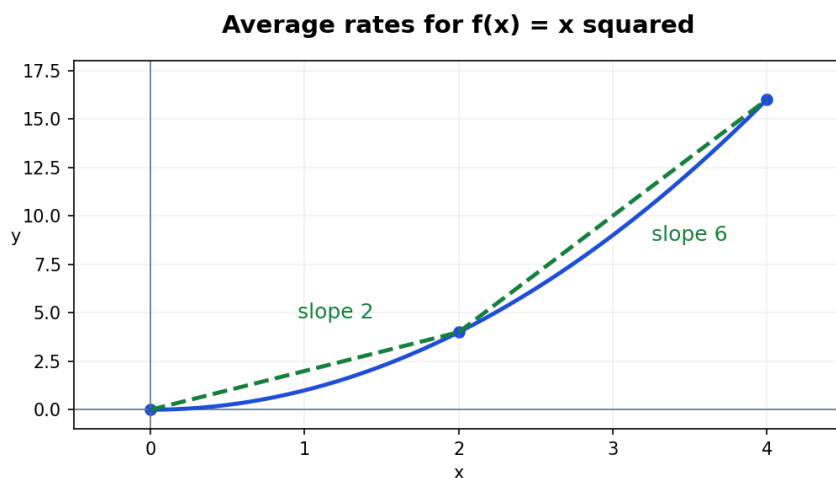


Solutions

1.1 Average Rate of Change

1.1.1. For $f(x) = x^2$, find the average rate of change on $[0, 2]$ and $[2, 4]$.

Solution. The rates are $(4 - 0)/2 = 2$ and $(16 - 4)/2 = 6$. Later secants are steeper on these positive intervals.



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Secant slopes on $[0, 2]$ and $[2, 4]$ are 2 and 6.

1.1.2. A car travels from mile marker 12 to mile marker 72 in 1.5 hours. Find average velocity.

Solution. The rate is $(72 - 12)/1.5 = 40$ miles per hour in the increasing-marker direction.

1.1.3. For $f(x) = 3x - 7$, show the average rate on any interval is 3.

Solution. $[3b - 7 - (3a - 7)]/(b - a) = 3(b - a)/(b - a) = 3$ for distinct endpoints.

Solutions

1.2 Successive Differences and Degree

1.2.1. Inputs 0, 1, 2, 3 give outputs 0, 1, 4, 9. Find the first and second differences.

Solution. First differences are 1, 3, 5; second differences are 2, 2. A quadratic model fits.

1.2.2. Can ordinary successive-difference degree tests be used unchanged at inputs 0, 1, 3, 6?

Solution. No. The input gaps are unequal. Equal-spacing assumptions fail; use divided differences or fit with the actual inputs.

1.2.3. Outputs 1, 8, 27, 64, 125 occur at inputs 1, 2, 3, 4, 5. Find the third differences.

Solution. First differences are 7, 19, 37, 61; second differences are 12, 18, 24; third differences are 6, 6. This agrees with a cubic.

Solutions

1.3 Zeros and Multiplicity

1.3.1. List zeros and multiplicities of $(x - 4)^3(x + 1)^2$.

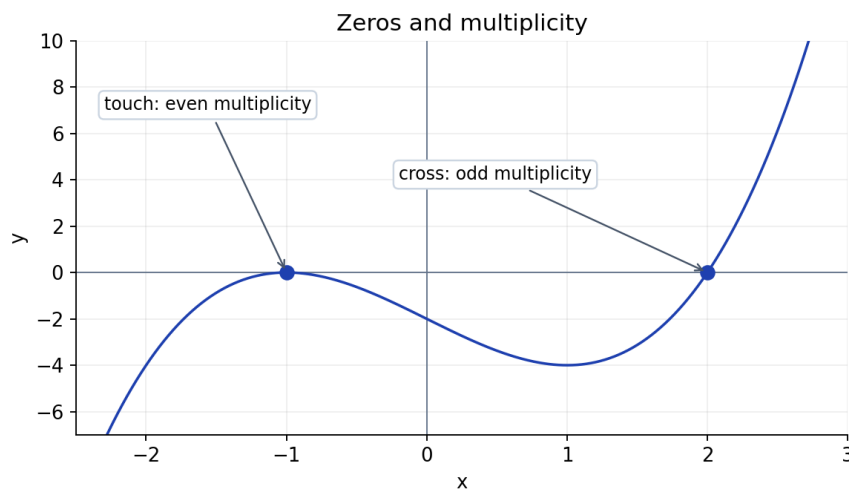
Solution. 4 has multiplicity 3 and -1 has multiplicity 2. The graph crosses at 4 and touches at -1 .

1.3.2. Write a monic quadratic with zeros -2 and 5.

Solution. $f(x) = (x + 2)(x - 5) = x^2 - 3x - 10$.

1.3.3. Use the graph of $y = (x + 1)^2(x - 2)$ to describe its zeros and whether it crosses or touches.

Solution. The zero -1 has multiplicity 2 and touches the axis. The zero 2 has multiplicity 1 and crosses.



-1 has multiplicity 2; 2 has multiplicity 1.

Solutions

1.4 End Behavior

1.4.1. Describe the ends of $f(x) = 5x^6 - x^2 + 2$.

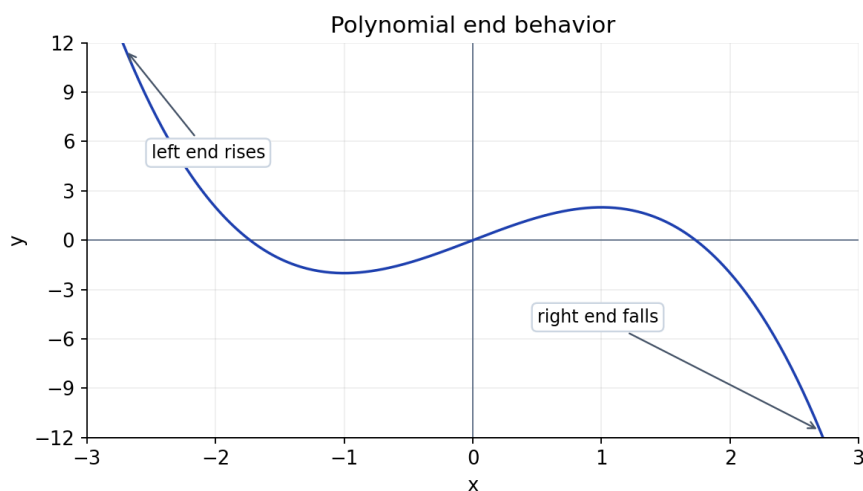
Solution. The degree is even and the leading coefficient is positive, so both ends rise toward ∞ .

1.4.2. Describe the ends of $g(x) = -x^5 + 2x^2$.

Solution. The negative odd-degree leading term makes the left end rise and the right end fall.

1.4.3. Use the graph of $y = -x^3 + 3x$ to identify its end behavior.

Solution. The negative cubic leading term means $y \rightarrow \infty$ as $x \rightarrow -\infty$, and $y \rightarrow -\infty$ as $x \rightarrow \infty$.



The left end tends to positive infinity; the right end tends to negative infinity.

Solutions

1.5 Vertical Asymptotes and Holes

1.5.1. Simplify $(x^2 - 4)/(x - 2)$ and state the hole.

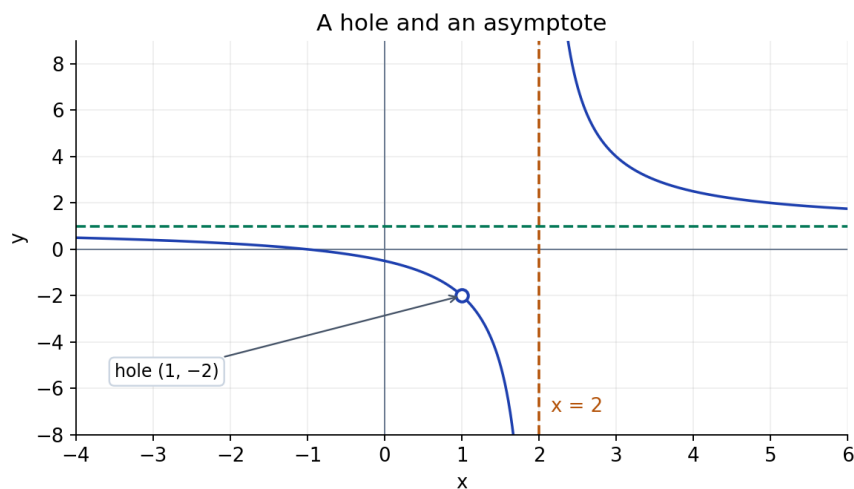
Solution. The reduced expression is $x + 2$, with $x \neq 2$. The hole is $(2, 4)$.

1.5.2. Find the excluded input and vertical asymptote of $1/(x + 3)$.

Solution. $x = -3$ is excluded and is a vertical asymptote.

1.5.3. For the graphed function $f(x) = (x^2 - 1)/((x - 1)(x - 2))$, state the hole and vertical asymptote.

Solution. Reducing gives $(x + 1)/(x - 2)$, with $x \neq 1, 2$. The hole is $(1, -2)$ and the vertical asymptote is $x = 2$.



The hole is $(1, -2)$; the vertical asymptote is $x = 2$.

Solutions

1.6 End Behavior of Rational Functions

1.6.1. Find the horizontal asymptote of $(2x + 1)/(x^2 + 4)$.

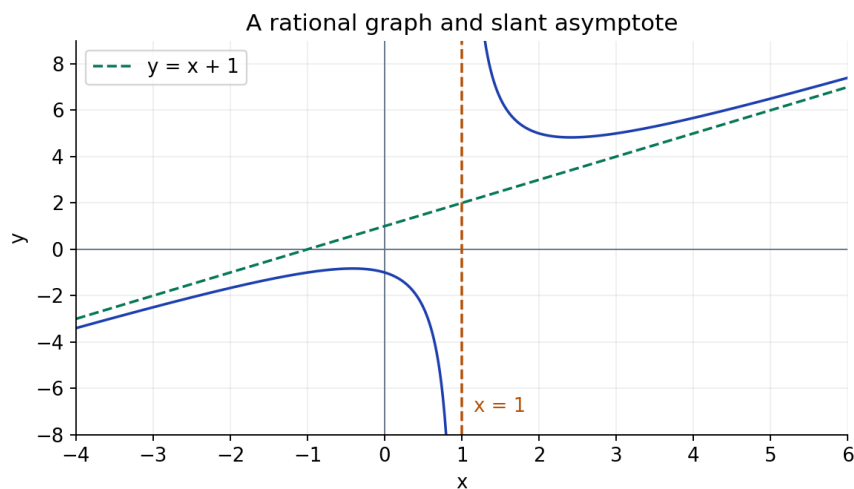
Solution. The numerator degree is smaller, so the horizontal asymptote is $y = 0$.

1.6.2. Find the horizontal asymptote of $(5x^3 - 1)/(2x^3 + x)$.

Solution. Equal degrees give $y = 5/2$.

1.6.3. Use the graph of $y = (x^2 + 1)/(x - 1)$ to identify its slant and vertical asymptotes.

Solution. Division gives $y = x + 1 + 2/(x - 1)$. The slant asymptote is $y = x + 1$; the vertical asymptote is $x = 1$.



The asymptotes are $x = 1$ and $y = x + 1$.

Solutions

1.7 Transformations of Functions

1.7.1. Describe $y = 3\sqrt{x+1} - 2$ relative to $y = \sqrt{x}$.

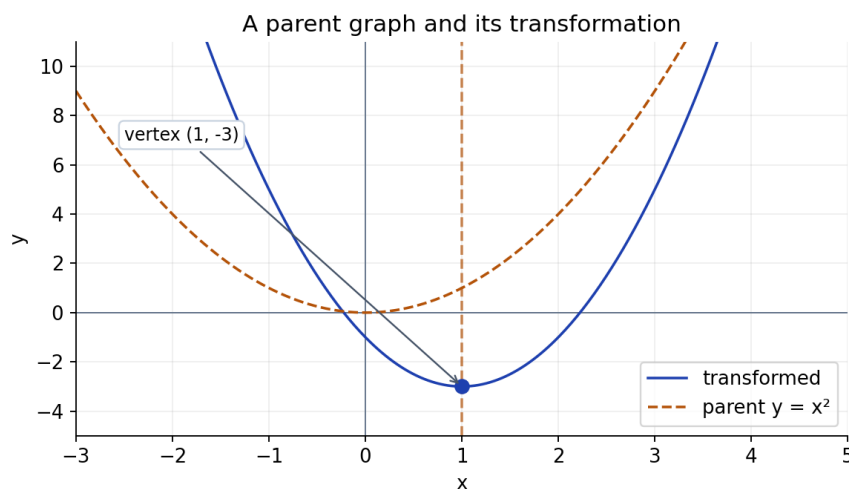
Solution. Shift left 1, stretch vertically by 3, and move down 2. The starting point is $(-1, -2)$.

1.7.2. Transform $y = x^2$ by reflecting across the x-axis and moving down 4.

Solution. The resulting equation is $y = -x^2 - 4$.

1.7.3. Compare the parent graph and transformed graph $y = 2(x-1)^2 - 3$. State its vertex and vertical scale.

Solution. The vertex moves to $(1, -3)$ and the vertical scale factor is 2. The graph opens upward and is narrower than $y = x^2$ at the same output height above its vertex.



The transformed vertex is $(1, -3)$, with vertical scale 2.