

Solutions

2.1 Constant Ratios and Model Choice

2.1.1. At inputs 0, 1, 2, 3, outputs are 2, 6, 18, 54. Give a model and predict the next output.

Solution. The ratio is 3, so $f(x) = 2 \cdot 3^x$ and $f(4) = 162$.

2.1.2. At inputs 0, 2, 4, outputs are 4, 16, 64. What is the per-unit growth factor?

Solution. A two-unit step multiplies by 4, so a one-unit step multiplies by $\sqrt{4} = 2$. A model is $f(x) = 4 \cdot 2^x$.

2.1.3. Why do outputs 1, 2, 4 at inputs 0, 1, 3 not establish a constant per-unit factor?

Solution. The input gaps are 1 and 2. Equal output ratios over unequal input changes represent different per-unit factors.

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2.2 Growth and Decay Factors

2.2.1. An initial amount of 100 grows by 10% per year. Find the amount after three years.

Solution. $100(1.1)^3 = 133.1$ units.

2.2.2. A model is $P(t) = 80(0.93)^t$. State the percent change each time unit.

Solution. The factor is $1 - 0.07$, so this is a 7% decrease per time unit.

2.2.3. Why does increasing by 20% and then decreasing by 20% not restore the original value?

Solution. The combined factor is $1.2(0.8) = 0.96$, a net 4% decrease. The percentages use different starting amounts.

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2.3 Rescaling the Exponent

2.3.1. A quantity starts at 12 and triples every 2 days. Find its value after 4 days.

Solution. $P(t) = 12 \cdot 3^{t/2}$, so $P(4) = 108$.

2.3.2. Rewrite $100 \cdot 4^{t/2}$ using a per-unit base.

Solution. Because $4^{1/2} = 2$, the equivalent model is $100 \cdot 2^t$.

2.3.3. A model uses t in years: $P(t) = 50(1.08)^t$. Rewrite using months m .

Solution. Substitute $t = m/12$: $P(m) = 50(1.08)^{m/12}$. The monthly factor is $(1.08)^{1/12}$.

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2.4 Logarithms as Inverses

2.4.1. Evaluate $\log_5(1/25)$.

Solution. Since $5^{-2} = 1/25$, the logarithm is -2 .

2.4.2. State the domain of $g(x) = \ln(x - 4)$.

Solution. The argument must be positive: $x - 4 > 0$, so the domain is $(4, \infty)$.

2.4.3. Solve $\log_3 x = 4$.

Solution. Convert to exponential form: $x = 3^4 = 81$.

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2.5 Logarithm Properties

2.5.1. Condense $\log_2 x + \log_2 (x - 1)$ and state the original domain.

Solution. The result is $\log_2 (x(x - 1))$, retaining $x > 1$ because both original arguments must be positive.

2.5.2. Evaluate $\log_3 (1/27)$.

Solution. $1/27 = 3^{-3}$, so the value is -3 .

2.5.3. Is $\ln(x + 2) = \ln x + \ln 2$ a valid identity for $x > 0$?

Solution. No. The product rule applies to $2x$, not $x + 2$. For example, $x = 1$ gives $\ln 3$ on the left and $\ln 2$ on the right.

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2.6 Solving Exponential and Logarithmic Equations

2.6.1. Solve $3^{2x} = 27$.

Solution. $27 = 3^3$, so $2x = 3$ and $x = 3/2$.

2.6.2. Solve $\log_2(x - 3) = 4$.

Solution. The domain requires $x > 3$. Rewrite as $x - 3 = 16$, giving $x = 19$.

2.6.3. Solve $\ln x + \ln(x - 3) = \ln 4$ and reject invalid candidates.

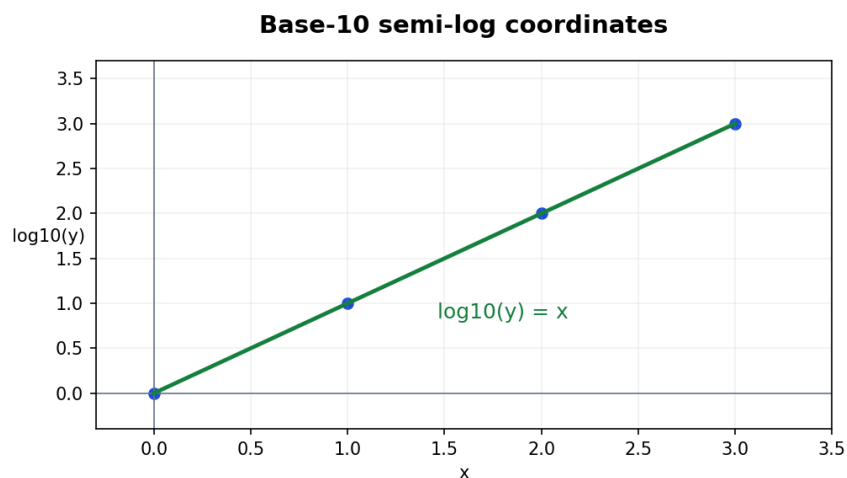
Solution. The original domain is $x > 3$. Combine logs to get $x(x - 3) = 4$, so $(x - 4)(x + 1) = 0$. Only $x = 4$ remains.

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2.7 Semi-Log Plots

2.7.1. At inputs 0, 1, 2, 3, outputs are 1, 10, 100, 1000. Find their base-10 logs and describe the semi-log pattern.

Solution. The log outputs are 0, 1, 2, 3, so the semi-log points lie on the line $\log_{10} y = x$.



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A straight line in these coordinates represents $y = 10$ to the power x .

2.7.2. Can a real semi-log plot use an output of 0?

Solution. No. The logarithm of zero is undefined in the real numbers.

2.7.3. If $\ln y = 3 - 0.2x$, find the per-unit factor in the original model.

Solution. Exponentiating gives $y = e^3(e^{-0.2})^x$, with decay factor $e^{-0.2} \approx 0.8187$.