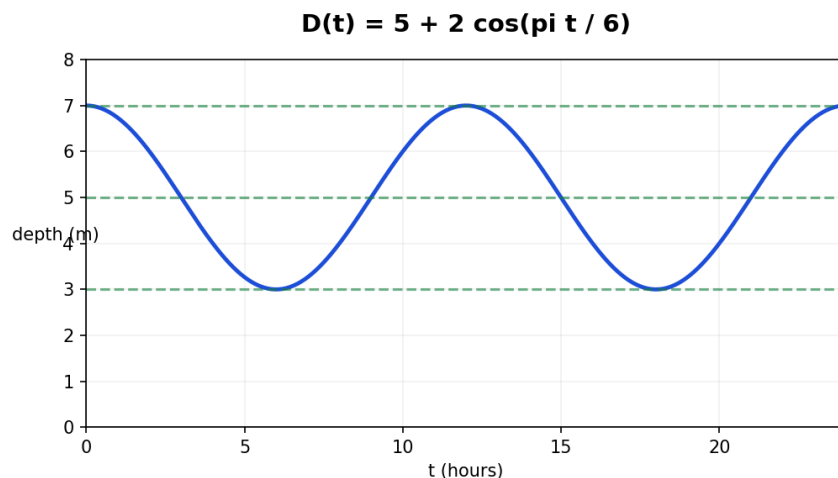


Solutions

3.1 Periodic Phenomena

3.1.1. For $D(t) = 5 + 2\cos(\pi t/6)$, find a repeat time and evaluate $D(0)$, $D(6)$, $D(12)$.

Solution. The period is 12. The depths are 7, 3, 7 meters respectively.



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Maximum depth = 7 m, minimum = 3 m, period = 12 h.

3.1.2. Does $f(0) = f(2)$ alone prove f has period 2?

Solution. No. A period requires $f(t+2) = f(t)$ for every relevant t . For example, $(t-1)^2$ has equal values at 0 and 2 but is not periodic.

3.1.3. A signal repeats every 0.25 seconds. Find its frequency in cycles per second.

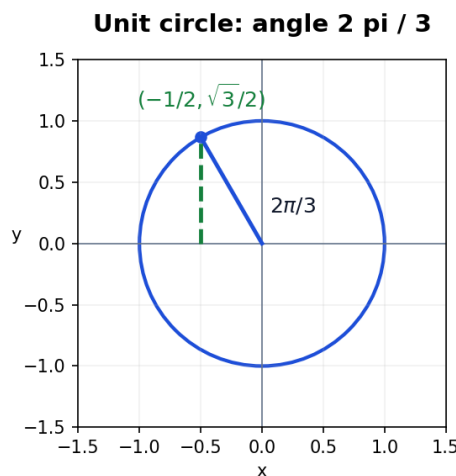
Solution. Frequency is the reciprocal of period: $1/0.25 = 4$ cycles per second.

Solutions

3.2 The Unit Circle

3.2.1. Find the unit-circle coordinates at $2\pi/3$.

Solution. The reference angle is $\pi/3$; quadrant II gives $(-1/2, \sqrt{3}/2)$.



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The terminal point is $(-1/2, \sqrt{3}/2)$; tangent is $-\sqrt{3}$.

3.2.2. Find $\sin(7\pi/6)$ and $\cos(7\pi/6)$.

Solution. Quadrant III makes both negative: $\sin = -1/2$ and $\cos = -\sqrt{3}/2$.

3.2.3. Give a positive coterminal angle for $-\pi/4$ in $[0, 2\pi)$.

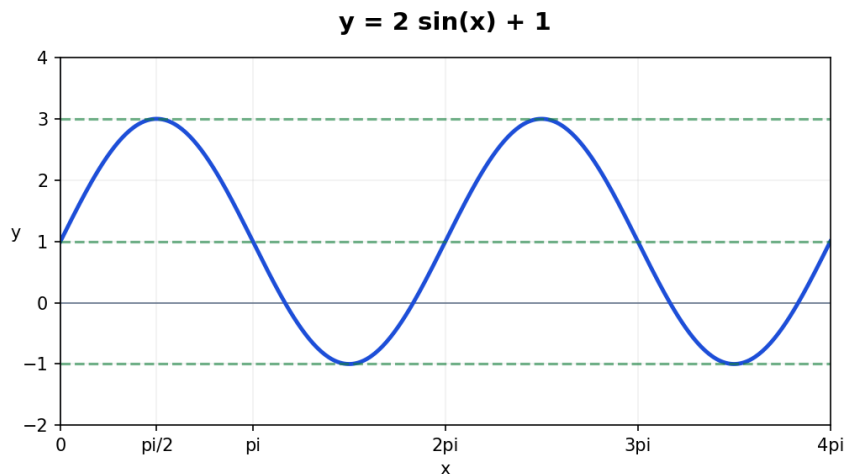
Solution. Add 2π to get $7\pi/4$.

Solutions

3.3 Sinusoidal Parameters

3.3.1. For $y = 2\sin x + 1$, find amplitude, period, midline, and range.

Solution. Amplitude 2, period 2π , midline $y = 1$, and range $[-1, 3]$.



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Amplitude = 2, midline = 1, period = 2π , range $[-1, 3]$.

3.3.2. Find the period of $y = -4\cos(3x)$.

Solution. The period is $2\pi/3$. The negative sign reflects the graph but does not change the period.

3.3.3. A sine model has period 8. Find a positive frequency coefficient.

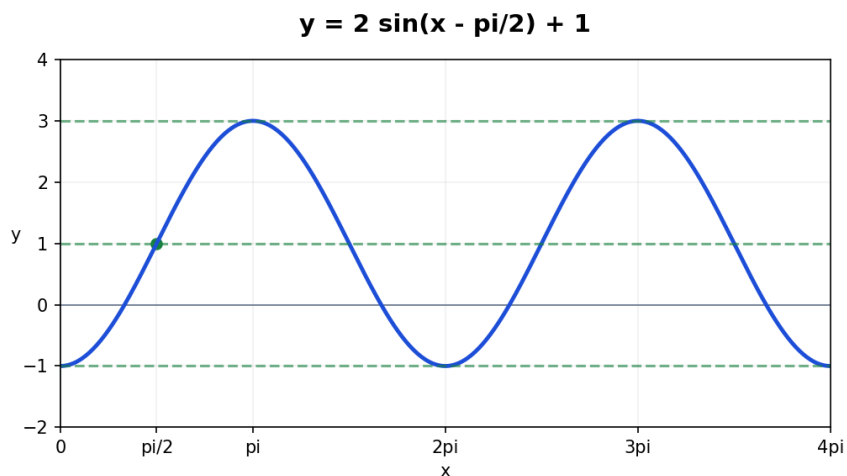
Solution. $B = 2\pi/8 = \pi/4$.

Solutions

3.4 Phase Shift

3.4.1. For $y = 2\sin(x - \pi/2) + 1$, find the shift and the value at $x = \pi/2$.

Solution. The shift is right $\pi/2$. The value is $2\sin 0 + 1 = 1$.



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The highlighted midline crossing is $(\pi/2, 1)$.

3.4.2. Factor the argument and find the shift of $y = \cos(3x + \pi)$.

Solution. It is $\cos(3(x + \pi/3))$, shifted left $\pi/3$.

3.4.3. Write a sine function of amplitude 1, period 2π , shifted right $\pi/3$.

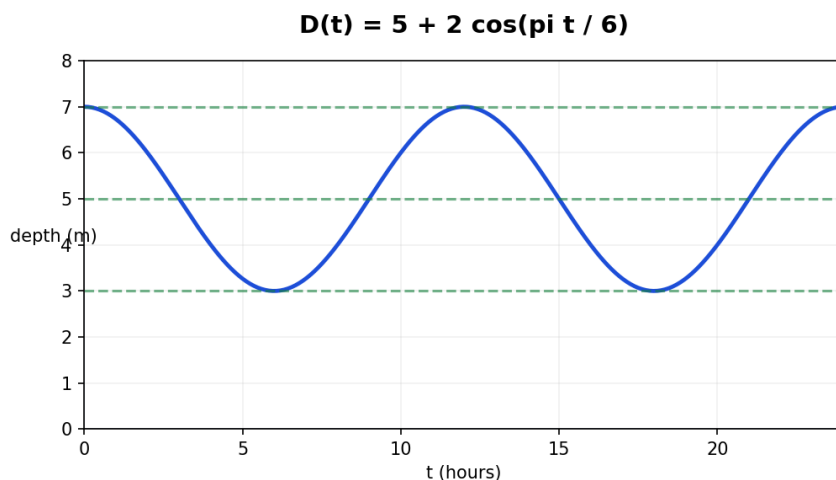
Solution. One answer is $y = \sin(x - \pi/3)$.

Solutions

3.5 Modeling with Sinusoids

3.5.1. Water depth is $D(t) = 5 + 2\cos(\pi t/6)$ meters with t in hours. Find maximum, minimum, and repeat time.

Solution. The maximum is 7 m, minimum 3 m, and period $2\pi/(\pi/6) = 12$ hours.



Maximum depth = 7 m, minimum = 3 m, period = 12 h.

3.5.2. For the depth model, find depth at $t = 3$ and $t = 6$ hours.

Solution. $D(3) = 5 + 2\cos(\pi/2) = 5$ m; $D(6) = 5 + 2\cos \pi = 3$ m.

3.5.3. A temperature cycles from 12 to 28 degrees every 24 hours and is maximal at $t = 0$. Give a model.

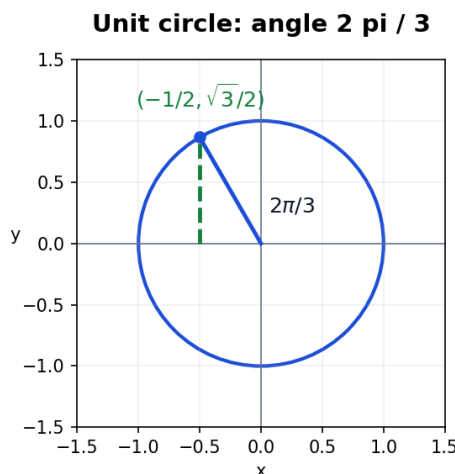
Solution. $T(t) = 20 + 8\cos(\pi t/12)$, with t in hours.

Solutions

3.6 Trigonometric Equations and Identities

3.6.1. Solve $\sin x = \sqrt{3}/2$ on $[0, 2\pi)$.

Solution. The reference angle is $\pi/3$. Sine is positive in quadrants I and II: $x = \pi/3, 2\pi/3$.



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The terminal point is $(-1/2, \sqrt{3}/2)$; tangent is $-\sqrt{3}$.

3.6.2. Solve $\sin x \cos x = 0$ on $[0, 2\pi)$.

Solution. Use the zero-product property. The solutions are $0, \pi/2, \pi, 3\pi/2$.

3.6.3. Solve $2\sin^2 x - \sin x = 0$ on $[0, 2\pi)$.

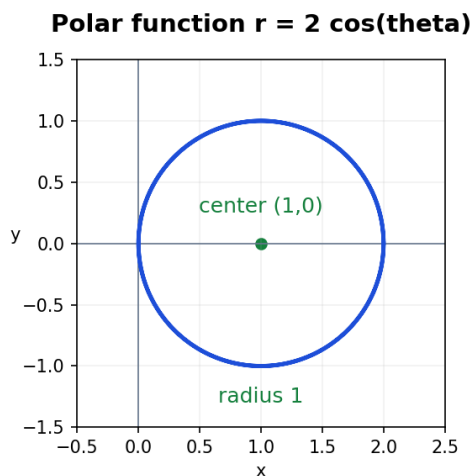
Solution. Factor $\sin x(2\sin x - 1) = 0$. From sine zero get $0, \pi$; from sine one-half get $\pi/6, 5\pi/6$. Do not divide by sine and lose the first pair.

Solutions

3.7 Polar Functions

3.7.1. For $r = 2\cos\theta$, find the radius and Cartesian point at $\theta = 2\pi/3$.

Solution. The radius is -1 . The Cartesian point is $(1/2, -\sqrt{3}/2)$, opposite the ray of angle $2\pi/3$.



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The Cartesian equation is $(x - 1)^2 + y^2 = 1$.

3.7.2. For $r = 3\sin\theta$, find the point at $\theta = \pi/2$.

Solution. The radius is 3, so the Cartesian point is $(0, 3)$.

3.7.3. Convert $r = 2\cos\theta$ to a Cartesian circle equation.

Solution. Multiply by r : $x^2 + y^2 = 2x$, hence $(x - 1)^2 + y^2 = 1$.