

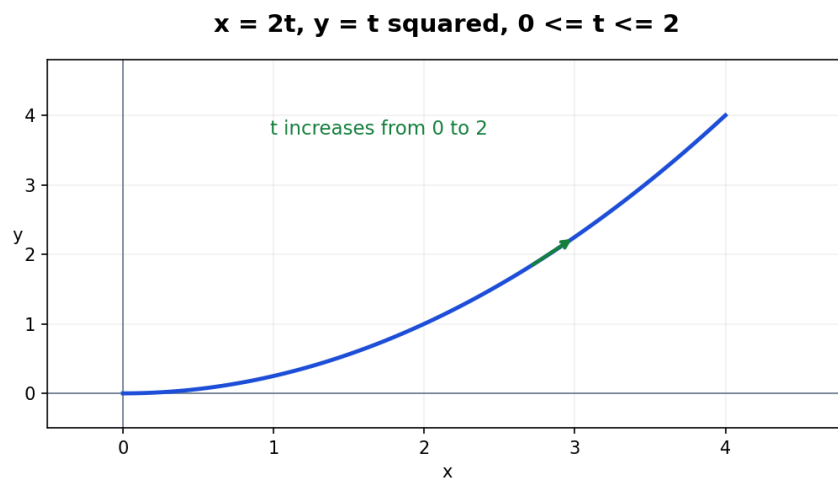
Solutions

4.1 Parametric Equations

Additional course content; not assessed on the AP Precalculus Exam.

4.1.1. For $x = 2t$, $y = t^2$, $0 \leq t \leq 2$, eliminate t and state the domain and direction.

Solution. $y = x^2/4$ for $0 \leq x \leq 4$, traced from $(0, 0)$ to $(4, 4)$ as t increases.



As t increases, the point moves from $(0,0)$ toward $(4,4)$.

4.1.2. For $x = 3\cos t$, $y = 2\sin t$, eliminate t .

Solution. The identity gives $x^2/9 + y^2/4 = 1$, an ellipse.

4.1.3. Why does eliminating t from $x = t^2$, $y = t^2$ not give the entire line $y = x$?

Solution. The parameter forces $x \geq 0$, so only the ray with $y = x \geq 0$ is traced.

Solutions

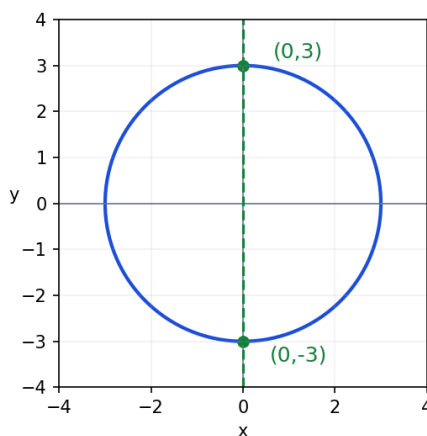
4.2 Implicitly Defined Curves

Additional course content; not assessed on the AP Precalculus Exam.

4.2.1. For $x^2 + y^2 = 9$, find all y -values when $x = 0$ and when $x = 3$.

Solution. At $x = 0$, $y = \pm 3$. At $x = 3$, $y = 0$. The first input fails the uniqueness requirement for a function.

Implicit curve: x squared + y squared = 9



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At $x = 0$ the two outputs are 3 and -3.

4.2.2. Solve $xy = 6$ for y and state the domain.

Solution. $y = 6/x$, with $x \neq 0$. This relation does define a function of x on that domain.

4.2.3. Does $x = y^2$ define y as a function of x over all its real points?

Solution. No. For $x > 0$ there are two outputs $\pm\sqrt{x}$. Restricting to $y \geq 0$ gives one branch.

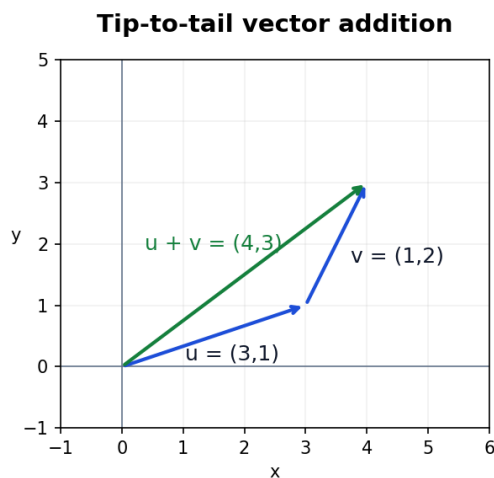
Solutions

4.3 Vectors

Additional course content; not assessed on the AP Precalculus Exam.

4.3.1. A displacement is $\mathbf{u} = (3, 1)$ followed by $\mathbf{v} = (1, 2)$. Find the resultant and its magnitude.

Solution. The resultant is $(4, 3)$ with magnitude 5.



The resultant from the origin is $(4,3)$, with magnitude 5.

4.3.2. Find a unit vector in the direction $(6, 8)$.

Solution. Its magnitude is 10, so a unit vector is $(3/5, 4/5)$.

4.3.3. Compute $(2, 1) \cdot (-1, 2)$ and interpret the result.

Solution. The dot product is $-2 + 2 = 0$, so the nonzero vectors are perpendicular.

Solutions

4.4 Matrix Operations

Additional course content; not assessed on the AP Precalculus Exam.

4.4.1. Compute $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Solution. Dot each row with the column: the result is $\begin{pmatrix} 4 \\ 10 \end{pmatrix}$.

4.4.2. Add $\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$ and $\begin{pmatrix} 3 & 2 \\ 4 & -2 \end{pmatrix}$.

Solution. Entrywise addition gives $\begin{pmatrix} 4 & 1 \\ 4 & 0 \end{pmatrix}$.

4.4.3. Why is a 2×3 matrix times a 2×2 matrix undefined?

Solution. The inner dimensions are 3 and 2, which do not agree.

Solutions

4.5 Determinants and Invertibility

Additional course content; not assessed on the AP Precalculus Exam.

4.5.1. Find the inverse of $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.

Solution. The inverse is $\begin{pmatrix} 1/2 & 0 \\ 0 & 1/3 \end{pmatrix}$. Multiplication gives the identity.

4.5.2. For what k is $\begin{pmatrix} k & 2 \\ 3 & 6 \end{pmatrix}$ singular?

Solution. Set determinant $6k - 6 = 0$ to obtain $k = 1$.

4.5.3. A planar linear transformation has determinant -3 . What happens to area and orientation?

Solution. Areas multiply by 3 and orientation reverses. The sign encodes orientation rather than negative physical area.

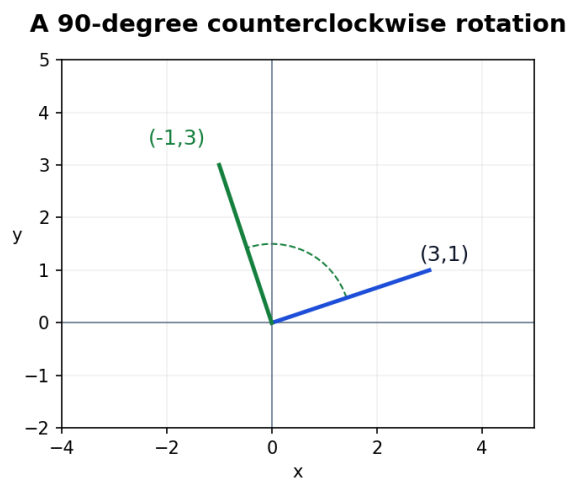
Solutions

4.6 Linear Transformations

Additional course content; not assessed on the AP Precalculus Exam.

4.6.1. Apply the 90° counterclockwise rotation matrix to the vector $(3, 1)$.

Solution. Matrix multiplication gives $(-1, 3)$. Its length remains $\sqrt{10}$.



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The matrix maps $(3,1)$ to $(-1,3)$ and preserves length.

4.6.2. Give the matrix of reflection across the x-axis and apply it to $(2, 5)$.

Solution. The matrix is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and the image is $(2, -5)$.

4.6.3. Explain why $T(x, y) = (x + 2, y)$ is not linear.

Solution. It sends $(0, 0)$ to $(2, 0)$. A linear transformation must send the zero vector to itself.