

Solutions

1.1 Conditional Statements

1.1.1. Write the converse of: If an integer is divisible by 4, it is even.

Solution. If an integer is even, it is divisible by 4. This is false; 6 is even but not divisible by 4.

1.1.2. Write the inverse of: If a triangle is equilateral, it is isosceles.

Solution. If a triangle is not equilateral, it is not isosceles. Under the inclusive definition of isosceles, the inverse is false: sides 5, 5, 6 give a counterexample.

1.1.3. Disprove: If $x^2 = 9$, then $x = 3$.

Solution. Take $x = -3$. Then $x^2 = 9$, but $x \neq 3$.

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1.2 Inductive and Deductive Reasoning

1.2.1. A sequence starts 2, 4, 8. Must its next term be 16?

Solution. No. Doubling suggests 16, but three terms do not uniquely determine a rule.

1.2.2. Use deduction: $AB = CD$, $CD = EF$, and $EF = 7$. Find AB .

Solution. By transitivity, $AB = EF = 7$.

1.2.3. Disprove: Every prime number is odd.

Solution. The prime number 2 is even. One counterexample is enough.

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1.3 Writing a Proof

1.3.1. Given $AB = CD$ and $CD = 2EF$, prove $AB = 2EF$.

Solution. Substitute $2EF$ for CD in $AB = CD$. This is the substitution property of equality.

1.3.2. Given $m\angle A + m\angle B = 90^\circ$ and $m\angle A = 35^\circ$, prove $m\angle B = 55^\circ$.

Solution. Substitute 35° and subtract from both sides: $m\angle B = 90^\circ - 35^\circ = 55^\circ$.

1.3.3. Why is a segment appearing horizontal insufficient to prove it perpendicular to a segment that appears vertical?

Solution. A sketch can be distorted. Use a right-angle marking, perpendicularity as a given, or slopes whose product is -1 when both slopes exist.