

Solutions

3.1 Exponential Models

3.1.1. Find the value of $P(t) = 500(1.04)^t$ at $t = 2$.

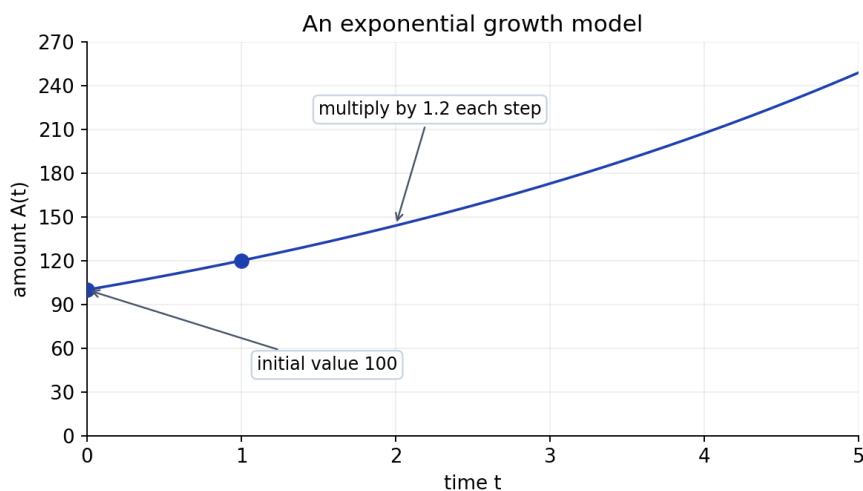
Solution. $P(2) = 500(1.04)^2 = 540.8$.

3.1.2. A quantity halves every 3 hours. Write its model when it starts at 120.

Solution. The number of half-life intervals is $t/3$, so $A(t) = 120(1/2)^{t/3}$.

3.1.3. From the graph of $A(t) = 100(1.2)^t$, identify the initial value and percentage growth per time unit.

Solution. $A(0) = 100$, so the initial value is 100. The factor $1.2 = 1 + 0.2$ gives 20% growth per time unit.



The model starts at 100 and grows by 20% per time unit.

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3.2 Logarithm Properties

3.2.1. Condense $\log_2 x + \log_2 (x - 1)$ and state the original domain.

Solution. The result is $\log_2 (x(x - 1))$, retaining $x > 1$ because both original arguments must be positive.

3.2.2. Evaluate $\log_3 (1/27)$.

Solution. $1/27 = 3^{-3}$, so the value is -3 .

3.2.3. Is $\ln(x + 2) = \ln x + \ln 2$ a valid identity for $x > 0$?

Solution. No. The product rule applies to $2x$, not $x + 2$. For example, $x = 1$ gives $\ln 3$ on the left and $\ln 2$ on the right.

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3.3 Solving and Domains

3.3.1. Solve $3^{2x} = 27$.

Solution. $27 = 3^3$, so $2x = 3$ and $x = 3/2$.

3.3.2. Solve $\log_2(x - 3) = 4$.

Solution. The domain requires $x > 3$. Rewrite as $x - 3 = 16$, giving $x = 19$.

3.3.3. Solve $\ln x + \ln(x - 3) = \ln 4$ and reject invalid candidates.

Solution. The original domain is $x > 3$. Combine logs to get $x(x - 3) = 4$, so $(x - 4)(x + 1) = 0$. Only $x = 4$ remains.